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The kinetic theory of a dense gas in the approximation of three-body interaction (hydrodynamic approximation)

F. B. BAIMBETOV and N. B. SHALTYKOV
The Kazakh State University, 480012, Alma-Ata, U.S.S.R.

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Abstract—Using the Bogolyubov method, the kinetic equations for dense gases in the presence of nonadditive three-body forces have been obtained. The account for the three-body interactions is shown to lead to the appearance, in the kinetic equations, of the collision integrals of the same order as that of the Choh–Uhlenbeck integral. Based on these equations the hydrodynamics of dense gases have been developed up to the n^2-order terms and general expressions have been obtained for the viscosity and thermal conductivity coefficients in the approximation of three-body interactions.

1. INTRODUCTION

A MOST exhaustive treatment of the kinetic theory of phenomena in dense gases up to the hydrodynamic approximation has been made in ref. [1], where the three-body collisions experienced by pairwise correlated particles are taken into account, i.e. the potential energy of the system is presented as a sum of the energies of two-body interactions. However, as pointed out in ref. [2], the requirement for the paired additivity of potential energy in dense gases is not met. Each molecule interacts with its immediate neighbourhood, and each interacting pair suffers effect from the neighbouring molecules. It has been shown in refs. [3, 4] that a nonadditive addition to the potential energy due to three-body interactions contributes strongly to the thermodynamic characteristics of noble gases. Many-body interactions play a substantial role in the phenomena on the surface [5, 6]. The forces of interaction between the molecules absorbed on the surface differ markedly from the forces of interaction between these very molecules in a gas [7, 8].

It has been shown in ref. [9] that an allowance for the three-body interaction potential allows a correct construction of the kinetic theory of dense gases and leads to the appearance, in the kinetic equations, of the terms of the same order as that of the Choh–Uhlenbeck collision integral.

The aim of this study is to construct the hydrodynamics of systems with three-body interactions, i.e. to derive the macroscopic transport equations and the expressions for heat conduction and viscosity coefficients which would allow a numerical analysis of these equations with the aid of a computer.

2. GENERAL MACROSCOPIC EQUATIONS

It is known that the state of a system at the hydrodynamic stage of its evolution is unambiguously characterized by such macroscopic quantities as the number density of particles, macroscopic gas velocity, kinetic temperature, and the density of internal energy. These quantities can be found with the aid of the reduced distribution functions F_1, F_2, F_3 by taking the momentum averages.

Determine the hydrodynamic functions of a dense gas [1, 9] as

$$n(\mathbf{q}, t) = \frac{1}{v}\int d\mathbf{p}\, F_1(\mathbf{p}, \mathbf{q}, t)$$

(number density of particles) (1)

$$mn(\mathbf{q}, t)\mathbf{u}(\mathbf{q}, t) = \frac{1}{v}\int d\mathbf{p}\, \mathbf{p}F_1(\mathbf{p}, \mathbf{q}, t)$$

(macroscopic flow velocity) (2)

$$\theta(\mathbf{q}, t) = \frac{2}{3n(\mathbf{q}, t)v}\int d\mathbf{p}\,\frac{(\mathbf{p}-m\mathbf{u})^2}{2m}F_1(\mathbf{p}, \mathbf{q}, t)$$

(kinetic temperature) (3)

$$n(\mathbf{q}, t)\varepsilon(\mathbf{q}, t) = n(\mathbf{q}, t)\{\tfrac{3}{2}\theta(\mathbf{q}, t) + \varepsilon^{\Phi}(\mathbf{q}, t)\} \quad (4)$$

(the density of internal energy which is comprised of the energy of chaotic motion of gas particles and the energy of interparticle interactions). In the approximation, which accounts for the three-body interaction, the quantity ε^{Φ} has the form

$$n(\mathbf{q}, t) = \varepsilon^{\Phi}\frac{1}{2v^2}\int d\mathbf{p}\int dx_2\Phi_{12}F_2 \times (x_1, x_2, t) + \frac{1}{6v^3}\int d\mathbf{p}\int dx_2\int dx_3 \times \Phi_{123}F_3(x_1, x_2, x_3, t), \quad (5)$$

where $x_i = (\mathbf{q}_i, \mathbf{p}_i)$, Φ_{12} is the potential energy of pairwise interaction, Φ_{123} is the potential energy of three-body interaction.

The kinetic equation, derived in ref. [9], in the three-

NOMENCLATURE

$\mathscr{A}^{(0)S}$	Boltzmann collision integral
D_{ij}	deformation tensor
F_s	distribution function of s particles in phase space
$f_{1,i}$	one-body distribution function of ith approximation with respect to density
f_{ij}	Mayer function
$\mathscr{F}^{(k)A,S}_{s,i}$	antisymmetric (A) and symmetric (S) terms of the expansion of functionals F_s of kth order in density and ith order in powers of relative distance
g_{ijk}	Mayer function analog for three-body interactions
$\mathbf{J}^k$	heat flux to random motion of gas particles
$\mathbf{J}^D$	heat flux corresponding to potential energy transport
m	mass of gas particles
n	number density of particles
$\mathbf{p}$	momentum of particles
$\hat{\mathbf{p}}$	momentum of random motion, $\mathbf{p}-m\mathbf{u}$
P	pressure
P^k_{ij}	pressure tensor due to thermal motion of molecules
P^{Φ}_{ij}	potential part of stress tensor
$\mathbf{q}_i$	coordinate of ith particle
R^{Φ}_1	heat generated by collisions
$\mathbf{r}_{ij}$	distance between ith and jth particles, $\mathbf{q}_i-\mathbf{q}_j$
$S^{(s)}_t$	evolution operator of s particles system
V	system volume
v	volume per particle, $1/n$
$\mathbf{u}$	hydrodynamic velocity of gas motion
x_i	coordinates of ith particle in phase space, $(\mathbf{p}_i, \mathbf{q}_i)$.

Greek symbols

β_i	ith virial coefficient
$\delta(x)$	Dirac delta-function
δ_{ij}	Kronecker delta
ε	total internal energy
ε^{Φ}	potential part of internal energy
$\eta^{(k)}_i$	ith viscosity coefficient in kth approximation with respect to density
θ	kinetic temperature
$\tau^{(i)}$	thermal conductivity coefficients in ith approximation with respect to density
ϕ_0	Maxwellian distribution
Φ_{ij}, Φ_{ijk}	pairwise and three-body interaction potentials of gas particles.

body interaction approximation takes on the form

$$\frac{\partial F_1}{\partial t}+\frac{\mathbf{p}_1}{m}\frac{\partial F_1}{\partial \mathbf{q}_1}=n\int\hat{\theta}_{12}S^{(2)}_{-\infty}(1,2)F_1F_1\,\mathrm{d}x_2$$
$$+n^2\int \mathrm{d}x_2\hat{\theta}_{12}\int_0^{\infty}\mathrm{d}\tau S^{(2)}_{-\tau}(1,2)$$
$$\times\int\{(\hat{\theta}_{13}+\hat{\theta}_{23})S^{(3)}_{-\infty}(1,2,3)$$
$$-S^{(2)}_{-\infty}(1,2)(\hat{\theta}_{13}S^{(2)}_{-\infty}(1,3)$$
$$+\hat{\theta}_{23}S^{(2)}_{-\infty}(2,3)\}F_1F_1F_1\,\mathrm{d}x_3$$
$$+n^2\int \mathrm{d}x_2\,\hat{\theta}_{12}\int_0^{\infty}\mathrm{d}\tau\int \mathrm{d}x_3\,S^{(2)}_{-\tau}(1,2)$$
$$\times\hat{\theta}_{123}S^{(3)}_{-\infty}(1,2,3)F_1F_1F_1+\frac{n^2}{2}$$
$$\times\int \mathrm{d}x_2\,\mathrm{d}x_3\hat{\theta}_{123}S^{(3)}_{-\infty}(1,2,3)F_1F_1F_1 \qquad (6)$$

where

$$\hat{\theta}_{ij}=\frac{\partial\Phi_{ij}}{\partial\mathbf{q}_i}\left(\frac{\partial}{\partial\mathbf{p}_i}-\frac{\partial}{\partial\mathbf{p}_j}\right),$$
$$\hat{\theta}_{123}=\frac{\partial\Phi_{123}}{\partial\mathbf{q}_1}\frac{\partial}{\partial\mathbf{p}_1}+\frac{\partial\Phi_{123}}{\partial\mathbf{q}_2}\frac{\partial}{\partial\mathbf{p}_2}+\frac{\partial\Phi_{123}}{\partial\mathbf{q}_3}\frac{\partial}{\partial\mathbf{p}_3}.$$

The remaining symbols are the same as in ref. [1].

Following ref. [1], the RHS of equation (6) can be expanded in power series with respect to the relative coordinates and quantities $S_{-\infty}\mathbf{r}_{12}$($S_{-\infty}$ is the product of time shift operators which depends on the relative coordinates $\mathbf{r}_{12}=\mathbf{q}_1-\mathbf{q}_2$). In this case, the following expansion for the functionals $F^{(0)}_2(x_1, x_2\,|\,F_1)$, $F^{(1)}_2(x_1, x_2\,|\,F_1)$, $F^{(0)}_3(x_1, x_2, x_3\,|\,F_1)$ is obtained

$$F^{(k)}_i=\mathscr{F}^{(k)S}_{i,0}+\mathscr{F}^{(k)S}_{i,1}+\mathscr{F}^{(k)A}_{i,1}+\ldots, \qquad (7)$$

where (k) indicates the order of expansion in powers of density; $i=2, 3$ are two- or three-body functions. The index after the comma indicates the order of expansion in powers of r_{12}; the symbols S and A characterize the properties of symmetry and antisymmetry under the permutation of particles 1 and 2. The expansion for $F^{(0)}_2(x_1, x_2\,|\,F_1)$ has exactly the same form as in ref. [1], while for $\mathscr{F}^{(1)S,A}_{2,i}(|\,F_1)$, $\mathscr{F}^{(0)S,A}_{3,i}(|\,F_1)$ the expansions are

$$\mathscr{F}^{(1)S}_{2,0}=\iiint \mathrm{d}\boldsymbol{\eta}_1\,\mathrm{d}\boldsymbol{\eta}_2\,\mathrm{d}\boldsymbol{\eta}_3\prod_{l=1}^{3}F_1(\mathbf{q}_1,\boldsymbol{\eta}_l)$$
$$\times\,\omega_0(\mathbf{r}_{12},\mathbf{p}_1,\mathbf{p}_2\,|\,\boldsymbol{\eta}_1,\boldsymbol{\eta}_2,\boldsymbol{\eta}_3),$$
$$\mathscr{F}^{(1)S}_{2,1}=\iiint \mathrm{d}\boldsymbol{\eta}_1\,\mathrm{d}\boldsymbol{\eta}_2\,\mathrm{d}\boldsymbol{\eta}_3\nabla^k_{\alpha}$$
$$\times\left\{\prod_{l=1}^{3}F_1(\mathbf{q}_1,\boldsymbol{\eta}_l)\right\}\omega_{1,k,\alpha}$$
$$\times(\mathbf{r}_{12},\mathbf{p}_1,\mathbf{p}_2\,|\,\boldsymbol{\eta}_1;\boldsymbol{\eta}_2;\boldsymbol{\eta}_3),$$

$$\mathscr{F}^{(1)S}_{2,2} = \iiint d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \nabla^k_\alpha \times \left\{ \nabla^j_\beta \prod_{l=1}^{3} F_1(\mathbf{q}_1, \boldsymbol{\eta}_l) \right\} \omega_{2,kj,\alpha\beta} \times (\mathbf{r}_{12}, \mathbf{p}_1, \mathbf{p}_2 \,|\, \boldsymbol{\eta}_1; \boldsymbol{\eta}_2; \boldsymbol{\eta}_3)$$

$$\mathscr{F}^{(1)A}_{2,1} = \frac{\partial}{\partial q_{1,\alpha}} \left(-\frac{r_{12\alpha}}{2} \mathscr{F}^{(1)S}_{2,0} \right),$$

$$\mathscr{F}^{(1)A}_{2,2} = \frac{\partial}{\partial q_{1,\alpha}} \left(-\frac{r_{12\alpha}}{2} \mathscr{F}^{(1)S}_{2,1} \right) \tag{8}$$

where

$$\omega_0(\mathbf{r}_{12}, \mathbf{p}_1, \mathbf{p}_2 \,|\, \boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \boldsymbol{\eta}_3) = \int_0^\infty d\tau\, S^{(2)}_{-\tau}(1,2) \int dx_3 \{ (\hat{\theta}_{13} + \hat{\theta}_{23}) \times S^{(3)}_{-\infty}(1,2,3) - S^{(2)}_{-\infty}(1,2)(\hat{\theta}_{13} S^{(2)}_{-\infty}(1,3) + \hat{\theta}_{23} S^{(2)}_{-\infty}(2,3)) + \hat{\theta}_{123} S^{(3)}_{-\infty}(1,2,3) \} \times \prod_{l=1}^{3} \delta(\mathbf{p}_l - \boldsymbol{\eta}_l). \tag{9}$$

The functions $\omega_{1,k,\alpha}$, $\omega_{2,kj,\alpha\beta}$ differ from expressions (4.56a and 4.56d) of ref. [1] by the presence of the term which is associated with the operator $\hat{\theta}_{123}$. These functions are cumbersome and are omitted to conserve space

$$\mathscr{F}^{(0)S}_{3,0} = \iiint d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \prod_{l=1}^{3} F_1(\mathbf{q}_1, \boldsymbol{\eta}_l) \gamma_0$$

$$\mathscr{F}^{(0)S}_{3,1} = \iiint d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \nabla^k_\alpha \left\{ \prod_{l=1}^{3} F_1(\mathbf{q}_1, \boldsymbol{\eta}_l) \right\} \gamma_{1,k,\alpha}$$

$$\mathscr{F}^{(0)S}_{3,2} = \iiint d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \nabla^k_\alpha \left\{ \nabla^j_\beta \prod_{l=1}^{3} F_1(\mathbf{q}_1, \boldsymbol{\eta}_l) \right\} \gamma_{2,kj,\alpha\beta}$$

$$\mathscr{F}^{(0)A}_{3,1} = \frac{\partial}{\partial q_{1\alpha}} \left(-\frac{r_{12\alpha}}{2} \mathscr{F}^{(0)S}_{3,0} \right),$$

$$\mathscr{F}^{(0)A}_{3,2} = \frac{\partial}{\partial q_{1\alpha}} \left(-\frac{r_{12\alpha}}{2} \mathscr{F}^{(0)S}_{3,1} \right) \tag{10}$$

where

$$\gamma_0 = S^{(3)}_\infty \prod_{l=1}^{3} \delta(\mathbf{p}_l - \boldsymbol{\eta}_l), \quad \gamma_{1,k,\alpha} = -\kappa^{(3)}_{k,\alpha} \gamma_0$$

$$\gamma_{2,kj,\alpha\beta} = \tfrac{1}{2} \kappa^{(3)}_{k,\alpha} \kappa^{(3)}_{j,\beta} \gamma_0,$$

$$\boldsymbol{\kappa}^{(3)}_1(1,2,3) = \frac{\mathbf{r}_{12}}{2} - (\tfrac{2}{3}\mathbf{r}_{13} - \tfrac{1}{3}\mathbf{r}_{23}) + S^{(3)}_\infty(\tfrac{2}{3}\mathbf{r}_{13} - \tfrac{1}{3}\mathbf{r}_{23}), \quad \boldsymbol{\kappa}^{(3)}_2 = \boldsymbol{\kappa}^{(3)}_1(2,1,3), \quad \boldsymbol{\kappa}^{(3)}_3(1,2,3) = -\tfrac{1}{6}(\mathbf{r}_{13} + \mathbf{r}_{23}) - \tfrac{1}{3} S^{(3)}_\infty(\mathbf{r}_{13} + \mathbf{r}_{23}).$$

Having integrated equation (6) over the momenta, the following continuity equation is obtained

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial q_{1,\alpha}}(n u_\alpha) = 0. \tag{11}$$

Multiplication of this equation by $\mathbf{p}$ and integration over the momenta yields the equation of motion in the form

$$mn \frac{Du_i}{Dt} = -\frac{\partial P^k_{i\alpha}}{\partial q_{1\alpha}} - \frac{\partial P^\Phi_{i\alpha}}{\partial q_{1\alpha}} \tag{12}$$

where $D/Dt = \partial/\partial t + u_\alpha(\partial/\partial q_{1\alpha})$, $P^k_{i\alpha}$ is the familiar expression for the pressure tensor resulting from the thermal motion of the molecules

$$P^\Phi_{i\alpha} = P^{\Phi_1}_{i\alpha} + P^{\Phi_2}_{i\alpha} + P^{\Phi_3}_{i\alpha} = -\frac{1}{v^2} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{r_{12\alpha}}{2} \{ \mathscr{F}^{(0)S}_{2,0} + \mathscr{F}^{(0)S}_{2,1} + \cdots \} - \frac{1}{v^3} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{r_{12\alpha}}{2} \{ \mathscr{F}^{(1)S}_{2,0} + \mathscr{F}^{(1)S}_{2,1} + \cdots \} - \frac{1}{2v^3} \int d\mathbf{p}_1\, dx_2\, dx_3 \frac{\partial \Phi_{123}}{\partial r_{12i}} \frac{r_{12,\alpha}}{2} \times \{ \mathscr{F}^{(0)S}_{3,0} + \mathscr{F}^{(0)S}_{3,1} + \cdots \}. \tag{13}$$

Multiplication of equation (6) by $p^2/2m$ and integration over the momenta gives the equation for the kinetic energy transport

$$n \frac{D}{Dt}(\tfrac{3}{2}\theta) + P_{i\alpha} D_{i\alpha} = -\frac{\partial}{\partial q_{1\alpha}} (J^k_\alpha + J^{\Phi I}_\alpha) + R^\Phi_1 \tag{14}$$

where $D_{i\alpha} = 1/2(\partial u_i/\partial q_{1\alpha} + \partial u_\alpha/\partial q_{1i})$ is the deformation tensor; J^k_α is the heat flux due to the random motion of the molecules

$$J^{\Phi I}_\alpha = J^{\Phi_1}_\alpha + J^{\Phi_2}_\alpha + J^{\Phi_3}_\alpha = -\frac{1}{v^2} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{\hat{p}_{1i} + \hat{p}_{2i}}{2m} \times \frac{r_{12\alpha}}{2} \{ \mathscr{F}^{(0)S}_{2,0} + \mathscr{F}^{(0)S}_{2,0} + \cdots \} - \frac{1}{v^3} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{\hat{p}_{1i} + \hat{p}_{2i}}{2m} \times \frac{r_{12\alpha}}{2} \{ \mathscr{F}^{(1)S}_{2,0} + \mathscr{F}^{(1)S}_{2,1} + \cdots \} - \frac{1}{2v^3} \int d\mathbf{p}_1\, dx_2\, dx_3 \frac{\partial \Phi_{123}}{\partial r_{12i}} \times \frac{\hat{p}_{1i} + \hat{p}_{2i} + \hat{p}_{3i}}{3m} \cdot \frac{r_{12\alpha}}{2} \{ \mathscr{F}^{(0)S}_{3,0} + \mathscr{F}^{(0)S}_{3,1} + \cdots \} \tag{15}$$

$$R^\Phi_1 = -\frac{1}{v^2} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{\hat{p}_{1i} - \hat{p}_{2i}}{2m} \times \{ \mathscr{F}^{(0)S}_{2,0} + \mathscr{F}^{(0)S}_{2,1} + \cdots \} - \frac{1}{v^3} \int d\mathbf{p}_1\, dx_2 \frac{\partial \Phi_{12}}{\partial r_{12i}} \frac{\hat{p}_{1i} - \hat{p}_{2i}}{2m} \times \{ \mathscr{F}^{(1)S}_{2,0} + \mathscr{F}^{(1)S}_{2,1} + \cdots \} - \frac{1}{2v^3} \int d\mathbf{p}_1\, dx_2\, dx_3 \times \frac{\partial \Phi_{123}}{\partial r_{12i}} \frac{2}{3} \frac{\hat{p}_{1i} - \hat{p}_{2i}}{m} \{ \mathscr{F}^{(0)S}_{3,0} + \mathscr{F}^{(0)S}_{3,1} + \cdots \} \tag{16}$$

is the heat released as a result of collisions.

In order to obtain the equation for the total internal energy, the equations for F_2 and F_3 are written as

$$\frac{\partial F_2}{\partial t}+\frac{\mathbf{p}_1}{m}\frac{\partial F_2}{\partial \mathbf{q}_1}+\frac{\mathbf{p}_2}{m}\frac{\partial F_2}{\partial \mathbf{q}_2}-\hat{\theta}_{12}F_2 = \frac{1}{v}\int \mathrm{d}x_3(\hat{\theta}_{13}+\hat{\theta}_{23}+\hat{\theta}_{123})F_3^{(0)} \quad (17)$$

$$\frac{\partial F_3^{(0)}}{\partial t}+\frac{\mathbf{p}_1}{m}\frac{\partial F_3^{(0)}}{\partial \mathbf{q}_1}+\frac{\mathbf{p}_2}{m}\frac{\partial F_3^{(0)}}{\partial \mathbf{q}_2}+\frac{\mathbf{p}_3}{m}\frac{\partial F_3^{(0)}}{\partial \mathbf{q}_3} = (\hat{\theta}_{12}+\hat{\theta}_{13}+\hat{\theta}_{23}+\hat{\theta}_{123})F_3^{(0)}. \quad (18)$$

Multiplying equation (17) by $(1/2v^2)\,\Phi_{12}$, equation (18) by $(1/6v^3)\,\Phi_{123}$, and integrating over $\mathbf{p}_1$, x_2 and x_3, yields

$$n\frac{\mathrm{D}}{\mathrm{D}t}\varepsilon^{\Phi_2}+\frac{\partial J_\alpha^{\Phi_2}}{\partial q_\alpha} = \frac{1}{2v^2}\int \mathrm{d}\mathbf{p}_1\,\mathrm{d}x_2\frac{\partial \Phi_{12}}{\partial r_{12\alpha}}\frac{\hat{p}_{1\alpha}-\hat{p}_{2\alpha}}{m}F_2 \quad (19)$$

$$n\frac{\mathrm{D}}{\mathrm{D}t}\varepsilon^{\Phi_3}+\frac{\partial J_\alpha^{\Phi_3}}{\partial q_{1\alpha}} = \frac{1}{6v^3}\int \mathrm{d}\mathbf{p}_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \times\frac{\partial \Phi_{123}}{\partial r_{12\alpha}}\frac{\hat{p}_{2\alpha}-\hat{p}_{3\alpha}}{m}F_3^{(0)}. \quad (20)$$

Adding together equations (14), (19) and (20) gives

$$n\frac{\mathrm{D}}{\mathrm{D}t}\varepsilon = -\frac{\partial J_\alpha}{\partial q_{1\alpha}}-P_{i\alpha}\mathrm{D}_{i\alpha} \quad (21)$$

where

$$\varepsilon = \frac{3}{2}\theta+\frac{1}{2v}\int \Phi_{12}F_2\,\mathrm{d}\mathbf{p}_1\,\mathrm{d}x_2 + \frac{1}{6v^2}\int \Phi_{123}F_3^{(0)}\mathrm{d}\mathbf{p}_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \quad (22)$$

$$J_\alpha = J_\alpha^k+J_\alpha^{\Phi\mathrm{I}}+J_\alpha^{\Phi\mathrm{II}}+J_\alpha^{\Phi\mathrm{III}}$$

$$J_\alpha^{\Phi\mathrm{II}} = \frac{1}{2v^2}\int \mathrm{d}\mathbf{p}_1\,\mathrm{d}x_2\Phi_{12}\frac{\hat{p}_{1\alpha}}{m}F_2,$$

$$J_\alpha^{\Phi\mathrm{III}} = \frac{1}{6v^3}\int \mathrm{d}\hat{p}_1\,\mathrm{d}x_2\,\mathrm{d}x_3\Phi_{123}\frac{\hat{p}_{1\alpha}}{m}F_3^{(0)}. \quad (23)$$

3. HYDRODYNAMIC APPROXIMATION

At the hydrodynamic stage of evolution, the characteristic times exceed the times of collisions, and it can be assumed that the one-particle distribution function depends on time only through its dependence on the macroscopic quantities. The departure from equilibrium in the approximation considered is caused by the inhomogeneity of the macroscopic quantities, so that the expansion parameter μ, on which the iteration procedure is based, will characterize the degree of the spatial variation of macroscopic quantities

$$\frac{1}{v}F_1(\mathbf{q},\mathbf{p}\,|\,n,\mathbf{u},\theta) = f_0(\mathbf{q},\mathbf{p}\,|\,n,\mathbf{u},\theta) + \mu f_1(\mathbf{q},\mathbf{p}\,|\,n,\mathbf{u},\theta)+\cdots \quad (24)$$

The hydrodynamic quantities should be expressed in terms of F_1 in a conventional way, therefore

$$n(\mathbf{q},t) = \int \mathrm{d}\mathbf{p}f_0 \quad (25)$$

$$n(\mathbf{q},t)\mathbf{u}(\mathbf{q},t) = \int \mathrm{d}\mathbf{p}\frac{\mathbf{p}}{m}f_0 \quad (26)$$

$$\frac{3}{2}n(\mathbf{q},t)\theta(\mathbf{q},t) = \int \mathrm{d}\mathbf{p}\frac{\hat{p}^2}{2m}f_0 \quad (27)$$

$$\int \mathrm{d}\mathbf{p}f_i = \int \mathrm{d}\mathbf{p}\frac{\mathbf{p}}{m}f_i = \int \mathrm{d}\mathbf{p}\frac{\hat{p}^2}{2m}f_i = 0, \quad i = 1,2,\ldots \quad (28)$$

The expansion of the hydrodynamic equations in powers of the parameter μ is of the form

$$\frac{\partial n}{\partial t} = \mu N^{(1)}+\mu^2 N^{(2)}+\cdots \quad (29)$$

$$\frac{\partial u_i}{\partial t} = \mu U_i^{(1)}+\mu^2 U_i^{(2)}+\cdots \quad (30)$$

$$\frac{\partial \theta}{\partial t} = \mu\Theta^{(1)}+\mu^2\Theta^{(2)}+\cdots \quad (31)$$

The expressions for f_0, f_1, f_2 are found from the expansion of the kinetic equation (6) in powers of the parameter μ. The zero approximation of this expansion is satisfied by the local equilibrium distribution

$$f_0(\mathbf{q},\mathbf{p}\,|\,n,\mathbf{u},\theta) = \frac{n(\mathbf{q},t)}{(2\pi m\theta(\mathbf{q},t))^{3/2}} \times\exp\left\{-\frac{[\mathbf{p}-m\mathbf{u}(\mathbf{q},t)]^2}{2m\theta(\mathbf{q},t)}\right\} \equiv n\phi_0(\hat{p}). \quad (32)$$

Noting the expansions (29)–(31), it is possible with the aid of $f_0(\mathbf{q},\mathbf{p})\,|\,n,\mathbf{u},\theta)$ in the first approximation over μ to obtain from equations (11), (12) and (21)

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial q_\alpha}(nu_\alpha) \quad (33)$$

$$\frac{\partial u_i}{\partial t} = -u_\alpha\frac{\partial u_i}{\partial q_\alpha}-\frac{1}{nm}\frac{\partial P}{\partial q_i} \quad (34)$$

$$\frac{\partial \varepsilon_0}{\partial t} = -u_\alpha\frac{\partial \varepsilon_0}{\partial q_\alpha}-\frac{P}{n}\frac{\partial u_\alpha}{\partial q_\alpha}. \quad (35)$$

Here the pressure P is defined by the equilibrium virial expansion [8, 9]

$$P = n\theta\left\{1-\frac{n}{2}\beta_1(\theta)-\frac{2n^2}{3}(\beta_2(\theta)+\tilde{\beta}_2(\theta))-\cdots\right\}, \quad (36)$$

where

$$\beta_1(\theta) = \frac{1}{1!}\int \mathrm{d}\mathbf{q}_2 f_{12}$$

$$\beta_2(\theta) = \frac{1}{2!}\int \mathrm{d}\mathbf{q}_2\,\mathrm{d}\mathbf{q}_3 f_{12}f_{13}f_{23}$$

$$\tilde{\beta}_2(\theta) = \frac{3}{4}\int \mathrm{d}\mathbf{q}_2\,\mathrm{d}\mathbf{q}_3(f_{12}+1)(f_{13}+1)(f_{23}+1)g_{123}$$

$$f_{ij} = \exp\left\{-\frac{\Phi_{ij}}{\theta}\right\}-1, \quad g_{123} = \exp\left\{-\frac{\Phi_{123}}{\theta}\right\}-1$$

are the Mayer functions.

$$n\varepsilon_0 = \frac{3}{2}n\theta\left\{1 + \frac{n}{3\theta}\int\Phi_{12}(f_{12}+1)\times\frac{d\mathbf{q}_1\,d\mathbf{q}_2}{V} + \frac{n^2}{3\theta}\int\Phi_{12}(f_{12}+1)\times f_{13}f_{23}\frac{d\mathbf{q}_1\,d\mathbf{q}_2\,d\mathbf{q}_3}{V} + \frac{n^2}{3\theta}\times\int\left[\Phi_{12}g_{123}(f_{12}+1)(f_{13}+1)(f_{23}+1) + \frac{\Phi_{123}}{3}(f_{12}+1)(f_{13}+1)(f_{23}+1)\times(g_{123}+1)\right]\frac{d\mathbf{q}_1\,d\mathbf{q}_2\,d\mathbf{q}_3}{V}\right\} \qquad (37)$$

is the energy density, which coincides with that in the equilibrium state theory [9].

In order to obtain the macroscopic equations in the next approximation in parameter μ, it is necessary to know the function f_1. It will be sought in the form of the series in powers of n

$$f_1 = f_{1,0} + nf_{1,1} + n^2 f_{1,2} + \cdots. \qquad (38)$$

The function $f_{1,0}$ coincides with that found in ref. [1] and has the form

$$f_{1,0} = -\phi_0(\hat{p})\left\{V^{(0)}(\hat{p})\frac{\hat{p}_\alpha}{m}\frac{\partial \ln\theta}{\partial q_\alpha} + W^{(0)}(\hat{p})\left(\frac{\hat{p}_\alpha\hat{p}_\beta}{m\theta} - \frac{\hat{p}^2\delta_{\alpha\beta}}{3m\theta}\right)D_{\alpha\beta}\right\}. \qquad (39)$$

Employing the iteration procedure described in ref. [1], the function $f_{1,1}$ is found in which, besides the effect of three-body collisions of pairwise correlated particles, the effective contribution of three-body interactions is taken into account

$$f_{1,1} = -\phi_0(\hat{p})\left\{V^{(1)}(\hat{p})\frac{\hat{p}_\alpha}{m}\frac{\partial \ln\theta}{\partial t} + \left[W_1^{(1)}(\hat{p})\left(\frac{\hat{p}_\alpha\hat{p}_\beta}{m\theta} - \frac{\hat{p}^2}{3m\theta}\delta_{\alpha\beta}\right)\times D_{\alpha\beta} + W_2^{(1)}(\hat{p})\delta_{\alpha\beta}D_{\alpha\beta}\right]\right\} \equiv -\phi_0(\hat{p})\times\left\{V_\alpha^{(1)}\frac{\partial\ln\theta}{\partial q_\alpha} + W_{\alpha\beta}^{(1)}D_{\alpha\beta}\right\}. \qquad (40)$$

4. CONTRIBUTION OF THREE-BODY INTERACTIONS

For a successive and rigorous allowance for the contribution of three-body interactions into the macroscopic equations of transport, the function f_1 should be found accurate to the terms of the order n^2, i.e. it is necessary to determine the function $f_{1,2}$. This will make it possible to calculate the contribution of triple correlations in addition to their effective contribution into the cross-section for pairwise interactions.

The function $f_{1,2}$ is sought in the form

$$f_{1,2} = -\phi_0(\hat{p})\left\{V_\alpha^{(2)}\frac{\partial\ln\theta}{\partial q_\alpha} + W_{\alpha\beta}^{(2)}D_{\alpha\beta}\right\}. \qquad (41)$$

The tensors $V_\alpha^{(i)}, W_{\alpha\beta}^{(i)}$ $(i = 0, 1, 2)$ satisfy the equations of the form

$$\mathscr{A}_1^{(0)S}(\,|\,\phi_0, \phi_0 V_\alpha^{(i)}) = -\phi_0 L_\alpha^{(i)}(\hat{p}) \qquad (42)$$

$$\mathscr{A}_1^{(0)S}(\,|\,\phi_0, \phi_0 W_{\alpha\beta}^{(i)}) = -\phi_0 M_{\alpha\beta}^{(i)}(\hat{p}) \qquad (43)$$

where $\mathscr{A}_1^{(0)S}$ is an ordinary Boltzmann collision integral, $L_\alpha^{(i)}, M_{\alpha\beta}^{(i)}$ are the familiar tensors. At $i = 0, 1$, the expressions for these tensors differ from those given on page 262 of ref. [1] by the terms which are interrelated with Φ_{123}. These are omitted to conserve space. For $i = 2$, these tensors have the form

$$L_\alpha^{(2)}(\hat{p}) = \frac{2}{3}(\beta_2' + \bar{\beta}_2')\frac{\hat{p}_\alpha}{m} - \frac{1}{(2\pi m\theta)^3}\times\exp\left\{\frac{\hat{p}^2}{2m\theta}\right\}\int d\boldsymbol{\eta}_1\,d\boldsymbol{\eta}_2\,d\boldsymbol{\eta}_3\times\left(\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta} - \frac{9}{2}\right)\exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\times\int d\mathbf{r}\,d\mathbf{p}_2\hat{\theta}_{12}\tilde{\omega}_{1,\alpha}$$
$$- \frac{1}{(2\pi m\theta)^3}\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}d\boldsymbol{\eta}_1\,d\boldsymbol{\eta}_2\,d\boldsymbol{\eta}_3\times\exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\sum_{k=1}^{3}\left(\frac{\hat{\eta}_k^2}{2m\theta} - \frac{3}{2}\right)\times\int d\mathbf{r}\,d\mathbf{p}_2\hat{\theta}_{12}\omega_{1,k,\alpha} - \frac{1}{2(2\pi m\theta)^3}$$
$$\times\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}\int d\boldsymbol{\eta}_1\,d\boldsymbol{\eta}_2\,d\boldsymbol{\eta}_3\times\exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\cdot\left(\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta} - \frac{9}{2}\right)\times\int dx_2\,dx_3\hat{\theta}_{123}\gamma_{1,\alpha} - \frac{1}{2(2\pi m\theta)^3}$$
$$\times\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}\int d\boldsymbol{\eta}_1\,d\boldsymbol{\eta}_2\,d\boldsymbol{\eta}_3\times\exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\cdot\sum_{i=1}^{3}\left(\frac{\hat{\eta}_i^2}{2m\theta} - \frac{3}{2}\right)\times\int dx_2\,dx_3\,\hat{\theta}_{123}\gamma_{1,i,\alpha} + \frac{1}{(2\pi m\theta)^3}$$
$$\times\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}\cdot\int d\boldsymbol{\eta}_1\,d\boldsymbol{\eta}_2\,d\boldsymbol{\eta}_3\times\exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\sum_{l=1}^{3}V_\alpha^{(1)}(\hat{\eta}_l)\times\int d\mathbf{r}\,d\mathbf{p}_2\,\hat{\theta}_{12}\,\omega_0 + \frac{1}{2(2\pi m\theta)^3}\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}$$

$$\times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\} \times \sum_{l=1}^{3} V_\alpha^{(1)}(\hat{\eta}_l)\cdot \int dx_2\, dx_3\, \theta_{123}\gamma_0. \quad (44)$$

$$M_{\alpha\beta}^{(2)}(\hat{p}) = \frac{2}{3}\left(\frac{p^2}{2m\theta}-\frac{3}{2}\right)\left[\frac{2}{3}(\beta_2+\tilde{\beta}_2)-C^{(1)}-\tilde{C}^{(1)}\right]\cdot\delta_{\alpha\beta}$$

$$-\frac{1}{(2\pi m\theta)^3}\exp\left\{+\frac{\hat{p}^2}{2m\theta}\right\}\left[\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\frac{\hat{\eta}_{1\beta}+\hat{\eta}_{2\beta}+\hat{\eta}_{3\beta}}{\theta}\right.$$

$$\times \int d\mathbf{r}\, d\mathbf{p}_2\, \hat{\theta}_{12}\tilde{\omega}_{1,\alpha} + \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\sum_{k=1}^{3}\frac{\hat{\eta}_{k,\beta}}{\theta}$$

$$\times \int d\mathbf{r}\, d\mathbf{p}_2\, \theta_{12}\omega_{1,j,\alpha} + \frac{1}{2}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\cdot\frac{\hat{\eta}_{1\beta}+\hat{\eta}_{2\beta}+\hat{\eta}_{3\beta}}{\theta}$$

$$\times \int dx_2\, dx_3\, \hat{\theta}_{123}\tilde{\gamma}_{1,\alpha} + \frac{1}{2}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\sum_{i=1}^{3}\frac{\hat{\eta}_{i,\beta}}{\theta}$$

$$\times \int dx_2\, dx_3\, \hat{\theta}_{123}\gamma_{1,i,\alpha} - \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\sum_{l=1}^{3} W_{\alpha\beta}^{(1)}(\hat{\eta}_l)$$

$$\times \int d\mathbf{r}\, d\mathbf{p}_2\cdot\hat{\theta}_{12}\omega_0 - \frac{1}{2}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \times \exp\left\{-\frac{\hat{\eta}_1^2+\hat{\eta}_2^2+\hat{\eta}_3^2}{2m\theta}\right\}\cdot\sum_{l=1}^{3} W_{\alpha\beta}^{(1)}(\hat{\eta}_l)$$

$$\left.\times \int d\mathbf{r}\, d\mathbf{p}_2\, dx_3\, \hat{\theta}_{123}\gamma_0\right]. \quad (45)$$

Knowing the structure of the function f_1, it is possible to find the stress tensors $P_{ij,1}^k$, $P_{ij,1}^\Phi$ and the energy flux density, which make it possible to write the hydrodynamic equations up to the second order in the parameter μ.

The stress tensor has the form

$$P_{ij,1} = P_{ij,1}^k + P_{ij,1}^\Phi = 2\eta_1(D_{ij}-\tfrac{1}{3}D_{\alpha\alpha}\delta_{ij})+\eta_2 D_{\alpha\alpha}\delta_{ij} \quad (46)$$

where

$$\eta_1 = \eta_1^{(0)}+n\eta_1^{(0)}+n^2\eta_1^{(2)}+\cdots \quad (47)$$

$$\eta_2 = n\eta_2^{(0)}+n^2\eta_2^{(2)}+\cdots \quad (48)$$

$$\eta_1^{(0)} = \frac{1}{15}\int d\mathbf{p}\frac{\mathbf{p}^4}{m^2\theta}\phi_0 W^{(0)}$$

$$\eta_1^{(1)} = \frac{1}{15}\int d\mathbf{p}\frac{p^4}{m^2\theta}W_1^{(1)}\phi_0(\hat{p}) - \frac{1}{10}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1\frac{\partial\Phi_{12}}{\partial r_\alpha}\frac{r^\beta}{2} \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\,\phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2)\phi_0(\hat{\eta}_3) \times \sum_{l=1}^{2} W^{(0)}(\hat{\eta}_l)\left\{\frac{\hat{\eta}_{l,\alpha}\hat{\eta}_{l,\beta}}{m\theta}-\frac{\hat{\eta}_l^2}{3m\theta}\delta_{\alpha\beta}\right\}\pi_0$$

$$\eta_1^{(2)} = \frac{1}{15}\int d\mathbf{p}\frac{p^4}{m^2\theta}W_1^{(2)}\phi(\hat{p}) - \frac{1}{10}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1\frac{\partial\Phi_{12}}{\partial\mathbf{r}_\alpha}\frac{r_\beta}{2} \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\,\theta_0(\hat{\eta}_1)\phi_0(\hat{\eta}_l) \times \sum_{l=2}^{2} W_1^{(1)}(\hat{\eta}_l)\left\{\frac{\hat{\eta}_{l,\alpha}\hat{\eta}_{l,\beta}}{m\theta}-\frac{\hat{\eta}_l^2}{3m\theta}\delta_{\alpha\beta}\right\}\pi_0$$

$$-\frac{1}{10}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{r}_1\, d\mathbf{p}_1\frac{\partial\Phi_{12}}{\partial r_\alpha}\frac{r_\beta}{2} \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\,\phi_0(\eta_1)\phi_0(\eta_2)\phi_0(\eta_3) \times \sum_{l=1}^{3} W^{(0)}(\hat{\eta}_l)\cdot\left(\frac{\hat{\eta}_{l,\alpha}\hat{\eta}_{l,\beta}}{m\theta}-\frac{\hat{\eta}_l^2}{3m\theta}\delta_{\alpha\beta}\right)\omega_0$$

$$-\frac{1}{20}\int d\mathbf{p}\, d\mathbf{p}_1\, d\mathbf{p}_2\int d\mathbf{r}\, d\mathbf{r}_2\frac{r_\alpha}{2}\frac{\partial\Phi_{12}}{\partial r_\beta} \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\,\phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2)\phi_0(\hat{\eta}_3) \times \sum_{l=1}^{3} W^{(0)}(\hat{\eta}_l)\left(\frac{\hat{\eta}_{l,\alpha}\hat{\eta}_{l,\beta}}{m\theta}-\frac{\hat{\eta}_l^2}{3m\theta}\delta_{\alpha\beta}\right)\gamma_0$$

$$\eta_2^{(1)} = \frac{1}{3}\int d\mathbf{p}\frac{p^2}{3m}\phi_0(p)W_2^{(1)}$$

$$\eta_2^{(2)} = \frac{1}{3}\int d\mathbf{p}\frac{p^2}{3m}\phi_0(p)W_2^{(2)} - \int d\mathbf{p}\, d\mathbf{p}_1\, d\mathbf{r}\frac{\partial\Phi_{12}}{\partial r_\beta}\frac{r_\beta}{2} \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\,\phi_0\phi_0\phi_0\sum_{l=1}^{3} W_2^{(1)}(\hat{\eta}_l)\pi_0. \quad (49)$$

Thus, the total stress tensor has the form of the Navier–Stokes one for viscous fluids with two viscosity coefficients. The contribution of three-body collisions is contained in $\eta_1^{(2)}$, $\eta_2^{(2)}$ and partially in $\eta_1^{(1)}$, $\eta_2^{(1)}$.

Substitution of the function f_1 into formula (23) yields the following expression for the heat flux density to the second order in the parameter μ

$$J_{i,1} = J_{i,1}^k + J_{i,1}^{\Phi\text{I}} + J_{i,1}^{\Phi\text{II}} + J_{i,1}^{\Phi\text{III}} = -\tau\frac{\partial\ln\theta}{\partial q_i} \quad (50)$$

where

$$\tau = \tau^{(0)} + n\tau^{(1)} + n^2\tau^{(2)} + \cdots \tag{51}$$

$$\tau^{(0)} = \frac{1}{3}\int d\mathbf{p}\frac{p^4}{2m^3}\phi_0(\hat{p})V^{(0)}(\mathbf{p})$$

$$\begin{aligned}\tau^{(1)} = {} & \frac{1}{3}\int d\mathbf{p}\frac{p^4}{2m^3}\phi_0(\hat{p})V^{(1)}(\hat{p}) \\ & -\frac{1}{3}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1 \frac{\partial\Phi_{12}}{\partial r_\beta}\frac{r_\alpha}{2} \\ & \times \frac{\hat{p}_\beta + \hat{p}_{1,\beta}}{2m}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2 \phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2) \\ & \times \sum_{l=1}^{2} V^{(0)}(\hat{\eta}_l)\frac{\hat{\eta}_{l,\alpha}}{m}\pi_0 \\ & + \frac{1}{6}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1 \Phi_{12}\frac{\hat{p}_\alpha + \hat{p}_{1\alpha}}{2m} \\ & \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, \phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2) \\ & \times \sum_{l=1}^{2} V^{(0)}(\hat{\eta}_l)\frac{\hat{\eta}_{l,\alpha}}{m}\pi_0 \end{aligned} \tag{52}$$

$$\begin{aligned}\tau^{(2)} = {} & \frac{1}{3}\int d\mathbf{p}\frac{p^4}{2m^3}\phi_0(\hat{p})V^{(2)}(\hat{p}) - \frac{1}{3}\int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1 \frac{\partial\Phi_{12}}{\partial r_\beta} \\ & \times \frac{r_{12\alpha}}{2}\frac{\hat{p}_\beta + \hat{p}_{1,\beta}}{2m}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, \phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2) \\ & \times \sum_{l=1}^{2} V^{(1)}(\hat{\eta}_l)\frac{\hat{\eta}_{l,\alpha}}{m}\pi_0 + \frac{1}{6} \\ & \times \int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1 \Phi_{12}\frac{\hat{p}_\alpha + \hat{p}_{1,\alpha}}{2m} \\ & \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, \phi_0(\hat{\eta}_1)\phi_0(\hat{\eta}_2) \sum_{l=1}^{2} V^{(1)}(\hat{\eta}_l) \\ & \times \frac{\hat{\eta}_{l,\alpha}}{m}\pi_0 - \frac{1}{3}\int d\mathbf{r}\frac{\partial\Phi_{12}}{\partial r_{12\alpha}}\cdot\frac{r_\beta}{2}\int d\mathbf{p}\, d\mathbf{p}_1 \\ & \times \frac{\hat{p}_\alpha + \hat{p}_{1,\alpha}}{2m}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\, \phi_0\phi_0\phi_0 \\ & \times \sum_{l=1}^{3} V^{(0)}(\hat{\eta}_l)\frac{\hat{\eta}_{l,\beta}}{m}\omega_0 + \frac{1}{6} \\ & \times \int d\mathbf{r}\, d\mathbf{p}\, d\mathbf{p}_1 \Phi_{12}\frac{\hat{p}_\alpha + \hat{p}_{1\alpha}}{2m}\int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3 \\ & \times \phi_0\phi_0\phi_0 \sum_{l=1}^{3} V^{(0)}(\hat{\eta}_l)\omega_0 + \frac{1}{18} \\ & \times \int d\mathbf{p}\, dx_2\, dx_3\, \Phi_{123}\frac{\hat{p}_{1,\alpha} + \hat{p}_\alpha}{2m} \\ & \times \int d\boldsymbol{\eta}_1\, d\boldsymbol{\eta}_2\, d\boldsymbol{\eta}_3\, \phi_0\phi_0\phi_0 \sum_{l=1}^{3} V^{(0)}\frac{\hat{\eta}_{l,\alpha}}{m}\gamma_0. \end{aligned}$$

In this approximation, the hydrodynamic equations take on the form

$$\begin{aligned} \frac{\partial n}{\partial t} &= -\frac{\partial}{\partial q_\alpha}(nu_\alpha) \\ \frac{\partial u_i}{\partial t} &= -\left(u_\alpha\frac{\partial u_i}{\partial q_\alpha} + \frac{1}{mn}\frac{\partial P}{\partial q_i} + \frac{1}{mn}\frac{\partial P_{i\alpha,1}}{\partial q_\alpha}\right) \\ \frac{\partial \varepsilon}{\partial t} &= -\left(u_\alpha\frac{\partial \varepsilon}{\partial q_\alpha} + \frac{1}{n}PD_{\alpha\alpha} + \frac{1}{n}P_{\alpha\beta,1}D_{\alpha\beta} + \frac{\partial J_{\alpha,1}}{\partial q_\alpha}\right) \end{aligned} \tag{53}$$

where $\varepsilon = \varepsilon_0 + \varepsilon_1^\Phi$, ε_0 is determined by formula (37) and

$$\begin{aligned} \varepsilon_1^\Phi = {} & \frac{1}{2}\iiint d\mathbf{p}_1\, dx_2\, \Phi_{12}\{[\mathscr{F}_{2,0}^{(0)S}(|f_0, f_1) \\ & + \mathscr{F}_{2,0}^{(1)S}(|f_0, f_1) + \cdots] \\ & + [\mathscr{F}_{2,1}^{(0)S}(|f_0) + \mathscr{F}_{2,1}^{(1)S}(|f_0) + \cdots] + \cdots\} \\ & + \frac{1}{6}\int d\mathbf{p}_1\, dx_2\, dx_3\, \Phi_{123}\{\mathscr{F}_{3,0}^{(0)S}(|f_0, f_1) \\ & + \mathscr{F}_{3,1}^{(0)S}(|f_0) + \cdots\}. \end{aligned} \tag{54}$$

5. CONCLUSION

Thus, the present paper summarizes the results of the theory of inhomogeneous dense gases with account for the three-body interactions up to the order n^2. It should be noted that the approach employed is more systematic than that given in ref. [1], since it consistently accounts for the contribution of three-body interactions into both the nondissipative and dissipative characteristics of a gas. The general expressions obtained for the thermal conductivity and viscosity coefficients can be easily transformed with the aid of the well-known technique of expansion in the orthogonal polynomials, in order to quantitatively estimate the contribution of three-body interactions. A numerical analysis of the transport phenomena within the framework of the approximation suggested will be the subject of further research.

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LA THEORIE CINETIQUE D'UN GAZ DENSE DANS L'APPROXIMATION DE L'INTERACTION A TROIS CORPS (APPROXIMATION HYDRODYNAMIQUE)

Résumé—En utilisant la méthode de Bogolyubov, on obtient les équations cinétiques des gaz denses en présence des forces non additives à trois corps. La prise en compte des interactions à trois corps conduit à l'apparition dans les équations cinétiques, des intégrales de collision du même ordre que celles de Choh–Uhlenbeck. Basée sur ces équations, l'hydrodynamique des gaz denses est développée jusqu'aux termes d'ordre n^2 et des expressions générales sont obtenues pour les coefficients de viscosité et de conductivité thermique, dans l'approximation des interactions à trois corps.

DIE KINETISCHE ENERGIE VON GASEN HOHER DICHTE UNTER VERWENDUNG DES NÄHERUNGSVERFAHRENS DER DREI-KÖRPER-WECHSELWIRKUNG (HYDRODYNAMISCHE NÄHERUNG)

Zusammenfassung—Unter Verwendung der Bogolyubov-Methode ergeben sich die kinetischen Gleichungen für Gase hoher Dichte in Gegenwart von nicht additiven Dreikörperkräften. Der Ansatz für die Drei-Körper-Wechselwirkung führt dazu, daß in den kinetischen Gleichungen Stoßintegrale auftreten, die dieselbe Ordnung wie das Choh–Uhlenbeck-Integral haben. Aufgrund dieser Gleichungen wurde die Hydrodynamik für Gase hoher Dichte einschließlich der Terme zweiter Ordnung (n^2) entwickelt, und es ergaben sich allgemeine Ausdrücke für die Viskositäts- und Wärmeleitfähigkeitskoeffizienten bei der Näherung der Drei-Körper-Wechselwirkung.

КИНЕТИЧЕСКАЯ ТЕОРИЯ ПЛОТНОГО ГАЗА В ПРИБЛИЖЕНИИ ТРЕХЧАСТИЧНЫХ ВЗАИМОДЕЙСТВИЙ (ГИДРОДИНАМИЧЕСКОЕ ПРИБЛИЖЕНИЕ)

Аннотация—Методом Боголюбова получены кинетические уравнения плотных газов в присутствии неаддитивных трехчастичных сил. Показано, что учет трехчастичных взаимодействий приводит к появлению в кинетических уравнениях интегралов столкновений того же порядка, что и интеграл Чо Уленбека. На основе этих уравнений построена гидродинамика плотных газов вплоть до членов порядка n^2 и получены общие выражения для коэффициентов вязкости и теплопроводности в приближении трехчастичных взаимодействий.